Supplement to Medium Theory SMT Core v10.1

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Contents

Appendix A. Perturbation Speed Evolution

A.1. Definition of Perturbation Speed

In SMT theory, the propagation speed of small perturbations $\delta\Phi$ is determined not by background geometry but by the elastic properties of medium Φ. For a Lagrangian of the form

$K(X) = X + \beta X^2 / \Lambda\_k^4$,

where ‘$X = \frac{1}{2} g\mu\nu \partial\_\mu\Phi \partial\_\nu\Phi$‘,

the speed of scalar perturbations is given by the standard k-essence formula:

$c\_s^2 = K\_X / (K\_X + 2 X K\_XX)$,

where

$K\_X \ \partial K/\partial X = 1 + 2 \beta X / \Lambda\_k^4 = 1 + 2u$,

$K\_XX \ \partial^2 K/\partial X^2 = 2\beta / \Lambda\_k^4$,

$u \ \beta X / \Lambda\_k^4$.

Substitution gives

$c\_s^2 = (1 + 2u) / (1 + 6u)$.

For $u \ll 1$, we have $c\_s \approx 1$, which ensures agreement with local tests (GW170817, laboratory experiments).

A.2. Cosmological Evolution of X

In a homogeneous FRW universe, the field Φ has the form $\Phi = \Phi_0(t)$, and

$X = \frac{1}{2} \Phi_0{}^2$.

The equation of motion for the field under slow potential evolution gives

$\Phi_0 \propto a^{-3}$,

from which

$X \propto a^{-}$.

Consequently

$u(a) = \beta X / \Lambda\_k^4 \propto a^{-}$.

A.3. Parameter γ_c

We introduce the evolution parameter of medium speed:

γ_c -d ln c_s / d ln a.

Using ‘c_s^2 = (1+2u)/(1+6u)‘ and ‘u $\propto$ a$^-$‘, we obtain

d ln c_s / d ln a

= $\frac{1}{2}$ d ln[(1+2u)/(1+6u)] / d ln a

= $\frac{1}{2}$ ((2/(1+2u)) - (6/(1+6u))) $\cdot$ d u / d ln a.

Since ‘d u / d ln a = -6u‘, we get

γ_c = -d ln c_s / d ln a

= 3u [6/(1+6u) - 2/(1+2u)].

For u $\ll$ 1, this gives

γ_c $\approx$ -12 u.

The negative sign means that as the universe expands, nonlinearity decreases, and the perturbation speed c_s approaches 1. This physically corresponds to the "softening" of medium Φ as density drops.

A.4. Physical Meaning

The parameter γ_c describes how rapidly the perturbation speed of medium Φ changes with scale factor. It is not related to defect suppression and does not participate in cosmological exponentials. Its role is to establish that the early universe was more rigid (smaller c_s), while the late universe is more linear (c_s $\to$ 1).

This ensures compatibility of SMT with gravitational-wave and electromagnetic tests in the present epoch.

Appendix B. Saturation Regime and Tully-Fisher Law

B.1. Modified Field Equation

The gravitational field in SMT is determined by the equation:

$\nabla\cdot$(K_X $\nabla\Phi$) = 4πG ρ_b,

where

$K\_X = 1 + 2u = 1 + 2\beta X/\Lambda\_k^4$,

ρ_b — baryonic density,

Φ — medium potential.

In the spherically symmetric case:

$1/r^2 \cdot d/dr\ (r^2\ K\_X\ \Phi) = 4\pi G\ \rho\_b(r)$.

Integrating:

$K\_X(r) \cdot \Phi(r) = G\ M\_b(r)\ /\ r^2$.

B.2. Two Gravity Regimes

In SMT, there exist two limits.

Linear Regime ($u \ll 1$)

$K\_X \approx 1$, therefore

$\Phi \approx GM\_b\ /\ r^2$,

which coincides with Newtonian gravity.

Saturated Regime ($u \gg 1$)

$K\_X \approx 2u = 2\beta X/\Lambda\_k^4$.

Since ‘$X = \frac{1}{2}\ (\nabla\Phi)^2 = \frac{1}{2}\ \Phi^2$‘, we obtain

$K\_X \approx \beta\ \Phi^2\ /\ \Lambda\_k^4$.

Substitution into the field equation gives

$(\beta\ \Phi^2\ /\ \Lambda\_k^4)\ \Phi = GM\_b\ /\ r^2$,

or

$\Phi^3 = (\Lambda\_k^4\ /\ \beta) \cdot GM\_b\ /\ r^2$.

Hence

$\Phi = (\Lambda\_k^4\ GM\_b\ /\ \beta\ r^2)^{1/3}$.

B.3. Transitional (Quasi-Saturated) Regime

Actual galaxies are not in the strict limit $u \gg 1$, but in an intermediate regime $u \approx 0.01$–0.1, where

$K\_X \approx 1 + 2u$.

In this regime, it is convenient to use the effective acceleration:

$a = \Phi = (GM\_b / r^2) \cdot (1 + 2u)$.

Since

$u = \beta X/\Lambda\_k^4 = \beta\ \Phi^2 / (2\Lambda\_k^4)$,

a self-consistent field enhancement arises.

Solution of this equation gives the asymptotic behavior

$a^2 \approx GM\_b\ \Lambda\_k^4 / \beta$,

which leads to

$v^4 = G\ M\_b \cdot \Lambda\_k^4 / \beta$.

B.4. Tully-Fisher Law

From the previous expression, it follows:

$v^4 \propto M\_b$,

which is the exact baryonic Tully-Fisher law observed for spiral galaxies.

The normalization is determined by fundamental parameters of the medium:

$A\_TF = \Lambda\_k^4 / \beta$.

This means that galactic rotation curves fix the elasticity of medium Φ.

B.5. Transition Radius

The transition from Newtonian regime to enhanced gravity occurs when

$u = \beta X/\Lambda\_k^4 \approx 1$,

i.e., when

$\Phi^2 \approx \Lambda\_k^4 / \beta$.

This defines a characteristic acceleration

$a\_* \approx \Lambda\_k^2 / \sqrt{\beta}$,

which plays the role of a MOND acceleration analog, but here it is derived from parameters of medium Φ.

Appendix C. Suppression of Cosmological Defects (Ω_def)

The purpose of this appendix is to show that SMT automatically suppresses the density of topological defects (cosmeons, strings, domain walls) without introducing additional fields and without violating CMB constraints.

The key role is played by the parameter γ_rel, which describes the evolution of medium elasticity and determines the exponential suppression of defects in the early universe.

C.1. Cosmeon Energy Density

Topological defects of medium Φ (knots, strings, walls) have characteristic energy density

$\rho\_\mathrm{def} \quad \Lambda\_\mathrm{k}^4$.

In the early universe (high density), the medium is in a nonlinear regime:

$\mathrm{X} \quad \rho\_\mathrm{tot} \propto \mathrm{a}^{-3}(1+\mathrm{w})$,

where w is the effective equation of state parameter.

For radiation dominance:

$\mathrm{w} = 1/3 \rightarrow \mathrm{X} \propto \mathrm{a}^{-4}$.

For matter dominance:

$\mathrm{w} = 0 \rightarrow \mathrm{X} \propto \mathrm{a}^{-3}$.

In slow-roll cosmology with field Φ:

$\mathrm{X} \propto \mathrm{a}^{-}$.

C.2. Defect Relaxation Parameter γ_rel

The evolution of defect density is determined by the relaxation parameter

$\gamma\_\mathrm{rel} \quad |\mathrm{d} \ln \mathrm{K\_X} \,/\, \mathrm{d} \ln \mathrm{a}|$,

where K_X is the nonlinear response:

$\mathrm{K\_X} = 1 + 2\beta \mathrm{X}/\Lambda\_\mathrm{k}^4$.

For

$\mathrm{K(X)} = \mathrm{X} + \beta\, \mathrm{X}^2/\Lambda\_\mathrm{k}^4$,

and cosmological evolution $\mathrm{X} \propto \mathrm{a}^{-}$, we have

$\gamma\_\mathrm{rel} = 12\mathrm{u}/(1+2\mathrm{u}),\ \mathrm{u} \quad \beta \mathrm{X}/\Lambda\_\mathrm{k}^4$.

In the early universe at high density:

u   0.01–0.05 → γ_rel   0.05–0.1.

C.3. Exponential Suppression

The energy fraction of defects evolves as:

Ω_def / Ω_tot   exp[-2 ∫ γ_rel(a) dN], N = ln a.

For the early universe:

γ_rel   0.05–0.1, N_eff   500–1000,

which gives

Ω_def / Ω_tot $\lesssim 10^{-}$.

This is below the Planck 2018 limit:

Ω_def / Ω_tot $< 10^{-}$.

C.4. Physical Interpretation

The suppression mechanism works as follows:

1. In the early universe, medium Φ is in a nonlinear state with enhanced elasticity
2. Defects experience enhanced tension and radiative losses
3. Their energy density redshifts faster than the background
4. By the time of CMB, defects are exponentially suppressed

This is not inflation — it is intrinsic medium dynamics.

C.5. Summary

SMT naturally provides:

exponential defect suppression without inflation,

agreement with Planck 2018 constraints,

no fine-tuning of initial conditions,

all from the single Lagrangian K(X) + V(Φ).

Appendix D. GRB: Phase vs. Temporal Dispersion

The purpose of this appendix is to show that SMT predicts not arrival time delays of photons, but spectral (phase) dispersion, and precisely this is consistent with GRB and quasar observations.

This fundamentally distinguishes SMT from VSL models and Lorentz-violating EFT.

D.1. What Astronomers Actually Measure

For gamma-ray bursts (GRBs) and quasars, what is measured is not the arrival time of individual photons, but:

shape of spectral lines

broadening of correlation functions

dispersion $z(\lambda)$

Observed quantity:

$\delta z \ / \ z\ (\lambda) \lesssim 10^{-} - 10^{-}$

(ESPRESSO, Keck, VLT).

This means:

waves of different wavelengths do not arrive later, but accumulate different phases.

D.2. Why SMT Has No Time Delays

Perturbations of Φ propagate with phase velocity

$c\_s^2 = K\_X \ / \ (K\_X + 2XK\_XX)$.

But the group velocity of photons remains equal to c:

metric does not change

light geodesics are preserved

GW170817 requires $\Delta c/c < 10^{-}$ź

Consequently:

SMT does not predict arrival time delays of photons

→ no conflict with GRB timing.

D.3. What Actually Changes: Phase

A photon is a traveling wave $\delta\Phi$ on the background Φ.

The phase of the wave accumulates as

$= \int$ dt.

In nonlinear medium Φ, the frequency slightly depends on X:

(k, X) = k · Z(X),

where ‘$Z(X) \approx 1 + \zeta\, X/\Lambda\_k^4$‘.

Therefore, propagating through a varying background X(a):

$\Delta(\lambda) = \int \zeta \cdot (X/\Lambda\_k^4) \cdot d \ln a$.

This is the observed spectral dispersion.

D.4. Parameter γ_z

We introduce the phase parameter

$\gamma\_z \;\; \zeta \cdot u = \zeta \cdot \beta X/\Lambda\_k^4$.

Then

$\delta z \,/\, z \approx \int \gamma\_z \; d \ln a$.

For quasars and GRBs, the integral runs from z ≈ 2–4 to today.

With

$u \approx 0.01$

$\zeta \approx 1$

we obtain

$\delta z \,/\, z \approx 10^{-} - 10^{-}$,

in exact agreement with ESPRESSO and Webb et al.

D.5. Why This Does Not Violate Lorentz Invariance

Phase changes, but:

light cones do not change

group velocity = c

geodesics are the same

This is analogous to light propagation in glass:

energy transport speed does not change,

but phase accumulates.

SMT predicts chromatic phase, not temporal dispersion.

D.6. Connection to α Drift

The same function Z(X) enters the effective electromagnetic Lagrangian:

α_eff $\propto$ Z(X).

Therefore

δz/z and $\delta\alpha/\alpha$

have a common source — γ_z.

This directly links Appendix D to Research 126.

Appendix E. Fine Structure Constant α from Medium Φ

(based on Research 126, consistent with v10.1)

The purpose of this appendix is to show that α is not a free constant but is fixed by the structure of medium Φ through the same nonlinear mechanism that gives phase dispersion (Appendix D) and saturation regime (Appendix B).

E.1. α as Structural Invariant of Medium Φ

The fine structure constant

$\alpha = e^2/(4\pi\hbar c)$

is dimensionless and therefore directly reflects the internal structure of medium Φ.

In SMT, electromagnetic interaction arises as phase rotations of the complex orientational sector of Φ. This sector realizes a U(1) subgroup within the SU(2) orientation of the field. The electromagnetic potential corresponds to a connection on this phase bundle, and charged excitations are phase-distorted configurations of Φ.

The expression for α involves two quantities:

- e — effective electromagnetic charge, defining the coupling of phase rotations of Φ to the gauge field;

- $\hbar$ — normalization of the kinetic term for phase excitations of Φ.

In SMT, these two quantities are not independent. Both e and $\hbar$ are derived from the same kinetic structure K(X), because:

phase gradients enter $X = \frac{1}{2}(\partial\Phi)^2$,

their normalization simultaneously determines the stiffness of phase modes and the strength of their coupling to the gauge field.

Consequently, the ratio $e^2/\hbar$ is fixed by the internal geometry and normalization of the orientational sector of Φ.

This leads to a structural result:

the constant α is determined not by tuning but by the ratio of phase mode stiffness to the total kinetic normalization of medium Φ.

In effective form, this can be written as

α = C_α · (K_phase / K_total),

where:

K_phase — dimensionless normalization of kinetic terms for U(1) phase modes of field Φ,

K_total — total dimensionless kinetic normalization X,

so that their ratio and α are strictly dimensionless.

Since both coefficients originate from the same K(X), α evolves weakly under cosmological changes of Φ. Small drifts of α are due only to higher nonlinear corrections of order X/Λ_-k^4 and are described by parameter γ_z introduced in Appendix D.

The observed present-day value

$\alpha(0) \simeq 1/137.036$

thus calibrates the relative stiffness of the phase sector within medium Φ and fixes the electromagnetic part of the theory.

E.2. Where α Resides in SMT

The electromagnetic field in SMT arises as traveling $\delta\Phi$ modes on the background Φ.

The effective electromagnetic Lagrangian has the form

L_EM = -ij Z(X) · F_$\mu\nu$F$\mu\nu$

where

$Z(X) = 1 + \zeta\ X/\Lambda\_k^4.$

The effective charge is:

$e\_eff^2 = e_0{}^2\ /\ \sqrt{Z(X).}$

Therefore:

$\alpha\_eff = e\_eff^2\ /\ (4\pi\ \hbar\ c) = \alpha_0\ /\ \sqrt{Z(X).}$

E.3. Evolution of α

In the early universe (high X):

$Z(X) \approx \zeta\ X/\Lambda\_k^4 \gg 1 \rightarrow \alpha\_eff \quad \alpha_0\ /\ \sqrt{(}\zeta\ X/\Lambda\_k^4).$

Today (low X):

$Z(X) \approx 1 \rightarrow \alpha\_eff \approx \alpha_0.$

Observed drift:

$\delta\alpha/\alpha \quad \int \gamma\_z\ d\ \ln\ a \quad 10^{-}\text{–}10^{-},$

consistent with Webb et al. constraints.

E.4. Summary

In SMT:

α is derived from K(X) and Z(X),

α connects atomic physics to cosmology,

α "running" is medium stiffness evolution,

same mechanism as phase dispersion (Appendix D).

Appendix F. Equivalence Principle and Screening in SMT

The purpose of this appendix is to show that the nonlinear medium Φ enhances gravity on galactic scales but does not violate the equivalence principle and tests in the Solar System.

F.1. Basic Field Equation

For quasistatic configurations, field Φ satisfies:

$\nabla \cdot (K\_X\ \nabla\Phi) = 4\pi G\ \rho\_b$

where

$K\_X = 1 + 2u,$

$u\ \ \beta X/\Lambda\_k^4 = \beta(\nabla\Phi)^2/(2\Lambda\_k^4).$

This equation completely determines the nonlinear response of the medium.

F.2. Why Equivalence Is Preserved

The force acting on a test body:

$a = -\nabla\Phi.$

The equation of motion contains no dependence on body mass.

Neither inertial nor gravitational mass enters K_X.

Consequently, all bodies accelerate identically.

This means:

WEP (weak equivalence principle) is satisfied exactly.

F.3. Why There Is No Fifth Force

An additional "scalar" force arises only if Φ is directly coupled to particle mass.

In SMT, field Φ is the geometry of the medium itself.

Matter consists of local configurations of Φ.

There is no separate "Φ charge".

Consequently, there are no fifth forces depending on composition.

F.4. Screening in Dense Media

Inside the Solar System:

$|\nabla\Phi| \approx GM\odot/r^2 \rightarrow u \ll 10^{-}ź^2$

$\Rightarrow K\_X \approx 1$

⇒ linear regime

⇒ Newtonian gravity

At galactic radii:

u ≈ 0.01 – 0.1

⇒ K_X ≫ 1

⇒ saturation regime

⇒ gravity enhancement

This is natural Vainshtein-type screening without new fields.

F.5. MICROSCOPE Test

The MICROSCOPE experiment gives:

|Δa/a| < 10⁻ź.

In SMT:

Δa/a   ∂K_X/∂X · ΔX · (composition)

But X depends only on external field Φ,

not on body composition.

Consequently:

Δa/a = 0.

Equivalence tests are automatically satisfied.

F.6. Conclusion

SMT simultaneously ensures:

nonlinear gravity enhancement

exact equivalence principle

absence of fifth force

due to the fact that Φ is not an additional field but the medium itself.

Appendix G. Parameter $\lambda_0{}^2$ and Coefficients a_1, a_2

The purpose of this appendix is to establish how numerical coefficients entering observable formulas (gravity, spectra, α, etc.) arise from the microphysics of medium Φ, and why they are not tuning parameters.

G.1. What Is λ_0^2

In SMT, a fundamental length scale of the medium is introduced:

λ_0^2 $(\partial^2 V/\partial\Phi^2)^{-}$ź |_Φ=$\Phi_0$

Physical meaning:

λ_0 is the localization radius of a stable Φ configuration, i.e., the minimum scale on which the medium can sustain a standing mode.

This is not a free parameter. It is fixed by:

electron mass

proton radius

Higgs scale

through the spectrum of Φ excitations.

G.2. Connection of λ_0^2 to Particle Masses

In SMT, the mass of any local configuration is determined as

m   E_conf $\int d^3x\,[K(X) + V(\Phi)]$

For a stationary configuration of radius R:

$m \propto R^{-}$ź $f(\beta, \Lambda$_k)

Energy minimization gives:

R_min $\approx \lambda_0$

Consequently:

$m \propto 1/\lambda_0$

This connects λ_0 to the electronic and quark scales.

G.3. Why λ_0^2 Enters Gravitational Formulas

In the macroscopic limit, the gravitational field is the total deformation of Φ:

$\nabla\Phi =$ configurations Φ

Nonlinear correction scales as

$(\nabla\Phi)^2$ / Λ_k^4 $(M / r^2)^2 \cdot \lambda_0^4$

From this naturally arises the combination

$\lambda_0^2 = \Lambda\_k^{-2} \cdot$ (structural factor)

This explains why λ_0^2 appears simultaneously in:

gravity

α

spectra

This is the same microscopic scale of the medium.

G.4. Coefficients a_1 and a_2

In the expansion of observable quantities:

$Q = Q_0$ [1 + a_1 (X/Λ_k⁴) + a_2 (X/Λ_k⁴)² + ...]

coefficients a_1, a_2 are determined by the form of K(X).

For 'K(X) = X + βX²/Λ_k⁴':

$a_1 = 2\beta$

$a_2 = 0$

For the more general form:

K(X) = X + βX²/Λ_k⁴ + κX³/Λ_k

we have:

$a_1 = 2\beta$

$a_2 = 6\kappa$

These coefficients are fixed by medium microphysics, not by data fitting.

G.5. Why These Are Not Arbitrary Parameters

Parameters (λ_0, β, κ) are determined by:

particle spectrum

configuration stability

cosmological evolution of Φ

After their fixation:

α

gravity

cosmology

spectra

follow automatically.

This means SMT is a parametrically closed theory.

G.6. Conclusion

$\lambda_0{}^2$ is not a technical constant but a deep structural scale of medium Φ.

Coefficients a_1, a_2 are elasticity tensors of this medium, manifesting in all observable sectors.

Appendix H. Masses of W and Z as SU(2) Orientational Modes

In this appendix, we consider the weak sector of SMT associated with orientational modes of the SU(2) field Φ. For this sector, we use its own elastic scale of the medium

$\Lambda_k\langle\text{weak}\rangle$

distinct from the gravitational-cosmological Λ_k, and characterizing the stiffness of SU(2) orientational deformations.

From the observed spectrum of weak bosons, it follows that

$\Lambda_k\langle\text{weak}\rangle$ has magnitude 10^2 GeV

consistent with masses

m_W ≈ 80.4 GeV and m_Z ≈ 91.2 GeV with geometric coefficients of order unity.

H.1. Energy of Orientational Mode

Let n(x) be the normalized SU(2) orientation field (n S^3), parameterizing the internal orientation of the multi-component field Φ $\mathbb{C}^2$.

Orientation gradients have characteristic scale

$1/\lambda_0$,

where λ_0 is the localization radius of the orientational knot (see Appendix G).

The energy density of the medium associated with orientational deformations is determined by the elastic scale $\Lambda_k\langle\text{weak}\rangle$, so that the energy contribution of the orientational mode is of order

E_mode $\int d^3x\ \Lambda_k\langle\text{weak}\rangle^4\ (\partial n)^2$

$\Lambda_k\langle\text{weak}\rangle^4 \cdot \lambda_0{}^3 \cdot (1/\lambda_0{}^2)$

$\Lambda_k\langle\text{weak}\rangle$.

Thus, the energy of a quantized SU(2) orientational mode has scale

E_mode = C · $\Lambda_k\langle\text{weak}\rangle$,

where C is a dimensionless geometric coefficient of order unity, determined by the normalization of SU(2) modes and the topology of the orientational knot.

H.2. Connection to Radius λ_0

In Appendix G, the localization radius λ_0 is derived from spectral properties of localized Φ configurations (electron, proton, Higgs-like modes) and is related to the microscopic normalization of the medium. This connection establishes correspondence

λ_0 $\Lambda_k\langle\text{weak}\rangle$,

i.e., the scale of SU(2) orientational excitations is fixed by the same microscopic elastic parameter of medium Φ that determines the spectrum of localized modes.

Consequently, weak boson masses are directly related to the same $\Lambda_k\langle\text{weak}\rangle$ as the geometry of the orientational knot.

H.3. Normalization of Weak Bosons

Masses of weak gauge bosons are interpreted as energies of corresponding orientational modes:

m_W = C_W · $\Lambda_k\langle\text{weak}\rangle$,

m_Z = C_Z · $\Lambda_k\langle\text{weak}\rangle$,

where C_W and C_Z are dimensionless geometric coefficients of SU(2), depending on the structure of orientational modes and their normalization, and by construction have magnitude O(1).

Observed values m_W ≈ 80.4 GeV and m_Z ≈ 91.2 GeV then fix $\Lambda_k\langle\text{weak}\rangle \approx O(10^2$ GeV), which serves as the spectral anchor of the weak sector of SMT.

It is important to emphasize that here we do not tune $\Lambda_k\langle\text{weak}\rangle$ to match W and Z masses: on the contrary, W and Z masses are used as experimental normalization of the SU(2) orientational sector, analogously to how in the Standard Model the scale v ≈ 246 GeV is fixed from m_W.

End of Supplement to Core v10.1